Triple and Quartic Interactions of Higgs Bosons in the Two-Higgs-Doublet Model with CP-violation

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Abstract

We consider the two-Higgs-doublet model with explicit CP-violation, where the effective Higgs potential is not CP-invariant at the tree-level. Three neutral Higgs bosons of the model are the mixtures of CP-even and CP-odd bosons which exist in the CP-conserving limit of the theory. The mass spectrum and tree-level couplings of the neutral Higgs bosons to gauge bosons and fermions are significantly dependent on the parameters of the Higgs boson mixing matrix. We calculate the Higgs-gauge boson, Higgs-fermion, triple and quartic Higgs self-interactions in the MSSM with explicit CP-violation in the Higgs sector and CP-violating Yukawa interactions of the third generation scalar quarks. In some regions of the MSSM parameter space substantial changes of the self-interaction vertices take place, leading to significant suppression or enhancement of the multiple Higgs boson production cross sections.

1 Introduction

General interest to the models with two (and more) Higgs doublets is maintained by the absence of a convincing argument in favor of only one generation of Higgs bosons when there are three known generations of fundamental fermions. Models with extended Higgs sector provide richer physical possibilities than the standard scheme with one doublet. One of them is the possibility to introduce CP-violation beyond the Cabibbo-Kobayashi-Maskawa (CKM) mechanism, by means of the Higgs boson exchange amplitudes with complex Higgs boson-fermion vertices. Complex couplings can be generated either spontaneously [1], when the vacuum expectation values of the Higgs fields are complex and couplings of the CP-invariant tree-level Higgs potential are real, or explicitly inserted [2] on the level of $SU(2)\times U(1)$-invariant potential terms, when the complex vacuum expectation values of scalar fields correspond to the minimum of hermitian potential with complex couplings, which is not CP-invariant (CP-invariance softly broken by the mass terms).

Various representations of the $SU(2)\times U(1)$-invariant two-doublet Higgs potentials have been considered in the literature. The two-doublet models with spontaneous CP-violation [1, 3] make use of the potential of general structure $-\mu^2\varphi^2+\lambda\varphi^4$ without the dimension two μ_{12}^2-terms. Models with explicit CP-violation use either the potential with trivial minimization ([2], see also [4]) or the potential with complex coupling μ_{12}^2 of the dimension two terms and complex couplings $\bar\lambda_5$, $\bar\lambda_6$ and $\bar\lambda_7$ in front of the dimension four potential terms [5, 6], similar to effective potential of the minimal supesymmetry (MSSM). Standard transformation (diagonalisation) procedure from the level of primary fields which are the components of scalar doublets in $SU(2)\times U(1)$-invariant potential terms ($SU(2)\times U(1)$ eigenstates), to the physical fields (mass eigenstates) of Higgs bosons should be consistently performed to respect the $SU(2)\times U(1)$ invariance and the minimization of the potential. We consider the diagonalisation for the two different two-Higgs-doublet potential forms in much details. A special case of the general two-Higgs-doublet model is represented by the Higgs sector of MSSM. Substantial radiative corrections to the Higgs boson masses and couplings are induced at the m_Z scale mainly by the third generation quarks t, b and the third generation scalar quarks [7]. In the special case of MSSM the multiparameter space of general two-Higgs-doublet model is significantly reduced, providing possibilities of much less ambiguous phenomenological predictions.

Phenomenological consequences of the CP-violating Higgs-third generation squark Yukawa interactions in the Higgs-fermion and the Higgs-gauge boson sectors have been considered in [5]. We focus mainly on the self-interactions of Higgs bosons. Experimental observation of the scalar boson signals should be followed by the verification of Higgs mechanism as the essence of the gauge boson and fermion mass generation. Self-interactions of the Higgs fields lead to untrivial structure of the vacuum state with nonzero (and possibly complex) field tensions, initializing the spontaneous breakdown of $SU(2) \times U(1)$ symmetry. Reconstruction of the Higgs self-interaction potential from the data on multiple (mainly double and triple) Higgs boson production cross sections [8] requires the experimental measurements of triple and quartic Higgs boson self-interaction vertices, which is nontrivial but valuable task for a future high luminosity colliders, such as LHC and TESLA.

In section 2 we discuss the diagonalisation of the Higgs potential, represented in two different forms, in the general two-Higgs-doublet model and discuss the MSSM limit of the model. In section 3 we introduce complex couplings of the $SU(2) \times U(1)$ invariant potential terms and discuss the diagonalisation by means of a rotation in the h, H, A space of CP-even and CP-odd Higgs bosons. In sections 4 and 5 we calculate the Higgs-gauge boson, Higgs-fermion and Higgs self-couplings in the MSSM with CP-violation.

2 Diagonalisation of the mass matrix in the general two-Higgs-doublet model

Two representations have been used for the two-doublet Higgs potential. The first representation [2, 4]

$$\begin{aligned} V(\varphi_1, \varphi_2) = \; & \lambda_1(\varphi_1^+\varphi_1 - \tfrac{v_1^2}{2})^2 + \lambda_2(\varphi_2^+\varphi_2 - \tfrac{v_2^2}{2})^2 \\ & +\lambda_3[(\varphi_1^+\varphi_1 - \tfrac{v_1^2}{2}) + (\varphi_2^+\varphi_2 - \tfrac{v_2^2}{2})]^2 \\ & +\lambda_4[(\varphi_1^+\varphi_1)(\varphi_2^+\varphi_2) - (\varphi_1^+\varphi_2)(\varphi_2^+\varphi_1)] \\ & +\lambda_5[\mathtt{Re}(\varphi_1^+\varphi_2) - \tfrac{v_1 v_2}{2}\mathtt{Re}(e^{i\xi})]^2 + \lambda_6[\mathtt{Im}(\varphi_1^+\varphi_2) - \tfrac{v_1 v_2}{2}\mathtt{Im}(e^{i\xi})]^2 \end{aligned} \quad (1)$$

where λ_i are real constants and the $SU(2)$ doublets $\varphi_{1,2}$ have the components

$$\varphi_1 = \{-iw_1^+, \frac{1}{\sqrt{2}}(v_1 + h_1 + iz_1)\}, \quad \varphi_1 = \{-iw_2^+, \frac{1}{\sqrt{2}}(v_2 + h_2 + iz_2)\}. \quad (2)$$

w is a complex field and z, $h_{1,2}$ are real scalar fields. At positive $\lambda_1, ...\lambda_6$ each term of the potential $V(\varphi_1, \varphi_2)$ is obviously positive and its zero minimum is achieved if the vacuum expectation values of $< \varphi_1 >$, $< \varphi_2 >$ are taken in the form

$$< \varphi_1 >= \frac{1}{\sqrt{2}}\{0, v_1\}, \quad < \varphi_2 >= \frac{1}{\sqrt{2}}\{0, v_2 e^{i\xi}\} \quad (3)$$

The λ_6 term is not CP-invariant. In the case of $\lambda_5 = \lambda_6$ (corresponding to the CP-conserving MSSM-like potential, see below) the last two terms in (1) form the modulo squared, and the phase ξ can be removed by the rotation of φ_1 or φ_2, which does not change the potential. In this section we will consider the case $\lambda_5 \neq \lambda_6$, ξ =0. Substitution of (2) to (1) gives a bilinear form of the mass term with mixed components $w, h_{1,2}, z$, which can be diagonalized by an orthogonal transformation of the fields in order to define the tree level masses of Higgs bosons. In the CP-conserving case the potential terms involving z_1, z_2 fields from the real parts of φ_1, φ_2 doublets and h_1, h_2 fields from the imaginary parts of φ_1, φ_2 doublets do not mix, so the mass terms are diagonalized by a separate two-dimensional rotations of the z_1, z_2 and the h_1, h_2 fields. The resulting spectrum of scalars consists of two charged $H^\pm$, three neutral h, H, A^0 scalar fields, and three Goldstone bosons G. This procedure is described in many papers (for instance, [4, 9]). The $w_{1,2}$ sector is diagonalized by the rotation of $w_1, w_2 \to H, G$

$$w_1^\pm = -H^\pm s_\beta + G^\pm c_\beta, \quad w_2^\pm = H^\pm c_\beta + G^\pm s_\beta \tag{4}$$

defined by the angle

$$\mathtt{tg}\beta = \frac{v_2}{v_1} \tag{5}$$

and leading to the massless G field and the field of massive charged Higgs boson $H^\pm$, $m^2_{H\pm} = \lambda_4(v_1^2 + v_2^2)/2$. The $z_{1,2}$ sector is diagonalized by the rotation $z_1, z_2 \to A^0, G'$ defined by the angle β and giving again one massless field G' and the field of CP-odd Higgs boson A^0 with the mass $m_A^2 = \lambda_5(v_1^2 + v_2^2)/2$. Finally, the h_1, h_2 sector is diagonalized by the rotation $h_1, h_2 \to h, H$ defined by the angle α

$$\mathtt{sin}2\alpha = \frac{2m_{12}}{\sqrt{(m_{11} - m_{22})^2 + 4m_{12}^2}}, \quad \mathtt{cos}2\alpha = \frac{m_{11} - m_{22}}{\sqrt{(m_{11} - m_{22})^2 + 4m_{12}^2}} \tag{6}$$

where

$$\begin{aligned} m_{11} &= \tfrac{1}{4}[4v_1^2(\lambda_1 + \lambda_3) + v_2^2\lambda_5] \\ m_{22} &= \tfrac{1}{4}[4v_2^2(\lambda_2 + \lambda_3) + v_1^2\lambda_5] \\ m_{12} &= \tfrac{1}{4}(4\lambda_3 + \lambda_5)v_1 v_2 \end{aligned}$$

giving two massive fields of CP-even Higgs bosons H, h with masses

$$m^2_{H,h} = m_{11} + m_{22} \pm \sqrt{(m_{11} - m_{22})^2 + 4m_{12}^2} \tag{7}$$

The diagonal mass matrix of scalar fields and the physical boson triple and quartic interaction vertices can be explicitly obtained by the substitution of the following expressions for λ_i to the potential $V(\varphi_1, \varphi_2)$ (1):

$$
\begin{aligned}
\lambda_1 =& \quad \tfrac{1}{2v^2}\tfrac{1}{c_\beta^2}[\tfrac{s_\alpha}{s_\beta}c_{\alpha-\beta}m_h^2 - \tfrac{c_\alpha}{s_\beta}s_{\alpha-\beta}m_H^2] + \tfrac{c_{2\beta}}{4c_\beta^2}\lambda_5 \\
\lambda_2 =& \quad \tfrac{1}{2v^2}\tfrac{1}{s_\beta^2}[\tfrac{c_\alpha}{c_\beta}c_{\alpha-\beta}m_h^2 + \tfrac{s_\alpha}{c_\beta}s_{\alpha-\beta}m_H^2] - \tfrac{c_{2\beta}}{4s_\beta^2}\lambda_5 \\
\lambda_3 =& \quad \tfrac{1}{2v^2}[-\tfrac{s_{2\alpha}}{s_{2\beta}}m_h^2 + \tfrac{s_{2\alpha}}{s_{2\beta}}m_H^2] - \tfrac{1}{4}\lambda_5 \\
\lambda_4 =& \quad \tfrac{2}{v^2}m_{H^\pm}^2 \\
\lambda_6 =& \quad \tfrac{2}{v^2}m_{A^0}^2
\end{aligned} \tag{8}
$$

where we used the notation $v^2 = v_1^2 + v_2^2$, $s_\alpha = \mathtt{sin}\alpha$, $c_\alpha = \mathtt{cos}\alpha$. Diagonalisation of the mass term takes place for arbitrary λ_5, which is a free parameter of the model not related to any Higgs boson mass.

The second representation of the Higgs potential

$$
\begin{aligned}
U(\varphi_1, \varphi_2) = \quad & -\mu_1^2(\varphi_1^+\varphi_1) - \mu_2^2(\varphi_2^+\varphi_2) - \mu_{12}^2(\varphi_1^+\varphi_2 + \varphi_2^+\varphi_1) \\
& +\bar{\lambda}_1(\varphi_1^+\varphi_1)^2 + \bar{\lambda}_2(\varphi_2^+\varphi_2)^2 + \bar{\lambda}_3(\varphi_1^+\varphi_1)(\varphi_2^+\varphi_2) \\
& +\bar{\lambda}_4(\varphi_1^+\varphi_2)(\varphi_2^+\varphi_1) + \tfrac{\bar{\lambda}_5}{2}[(\varphi_1^+\varphi_2)(\varphi_1^+\varphi_2) + (\varphi_2^+\varphi_1)(\varphi_2^+\varphi_1)]
\end{aligned} \tag{9}
$$

originates from the general SUSY action after the integration over Grassman variables and introduction of the soft SUSY-breaking terms (see [4]). It is easy to check that in the case of zero $\varphi_1^+\varphi_1$ phase the potentials (1) and (9) are equivalent if constants $\bar{\lambda}_i$, μ and λ_i are related by the formulae

$$
\begin{aligned}
\bar{\lambda}_1 = \lambda_1 + \lambda_3, \quad \bar{\lambda}_2 = \lambda_2 + \lambda_3, \quad \bar{\lambda}_3 = 2\lambda_3 + \lambda_4, \\
\bar{\lambda}_4 = -\lambda_4 + \frac{\lambda_5}{2} + \frac{\lambda_6}{2}, \quad \bar{\lambda}_5 = \frac{\lambda_5}{2} - \frac{\lambda_6}{2}
\end{aligned} \tag{10}
$$

and

$$
\mu_{12}^2 = \lambda_5\frac{v_1 v_2}{2}, \quad \mu_1^2 = \lambda_1 v_1^2 + \lambda_3 v_1^2 + \lambda_3 v_2^2, \quad \mu_2^2 = \lambda_2 v_2^2 + \lambda_3 v_1^2 + \lambda_3 v_2^2 \tag{11}
$$

Unlike the potential (1) where the minimization is obvious, the symbolic structure of (9) does not demonstrate evidently its minimum. The substitution of (2) to (9) gives linear terms in the component fields $z_{1,2}$, $h_{1,2}$ (or physical fields h, H, A) and unless some additional conditions to remove the linear terms are imposed, we are not in the minimum of the potential. So the equations (11) which set to zero the terms which are linear in component fields are the minimization conditions. The diagonalisation of $U(\varphi_1, \varphi_2)$ takes place for arbitrary $\bar{\lambda}_5$, which is linearly dependent on the μ_{12}^2 parameter in (11).

Inverse transformation (10) has the form

$$\lambda_1 = \bar\lambda_1 - \frac{\bar\lambda_3}{2} - \frac{\bar\lambda_4}{2} - \frac{\bar\lambda_5}{2} + \frac{\lambda_5}{2}, \quad \lambda_2 = \bar\lambda_2 - \frac{\bar\lambda_3}{2} - \frac{\bar\lambda_4}{2} - \frac{\bar\lambda_5}{2} + \frac{\lambda_5}{2} \tag{12}$$
$$\lambda_3 = \frac{\bar\lambda_3}{2} + \frac{\bar\lambda_4}{2} + \frac{\bar\lambda_5}{2} - \frac{\lambda_5}{2}, \quad \lambda_4 = -\bar\lambda_4 - \bar\lambda_5 + \lambda_5, \quad \lambda_6 = -2\,\bar\lambda_5 + \lambda_5$$

so masses of the CP-even scalars and their mixing angle α (6),(7) in the case of potential $U(\varphi_1, \varphi_2)$ can be easily obtained using

$$m_{11} + m_{22} = v_1^2\bar\lambda_1 + v_2^2\bar\lambda_2 + \frac{\mu_{12}^2}{s_{2\beta}}, \; m_{11} - \bar m_{22} = v_1^2\bar\lambda_1 - v_2^2\bar\lambda_2 - \mathtt{ctg}\,2\beta\mu_{12}^2, \tag{13}$$
$$2m_{12} \;=\; v_1 v_2(\bar\lambda_3 + \bar\lambda_4 + \bar\lambda_5) - \mu_{12}^2.$$

The diagonal form of $U(\varphi_1, \varphi_2)$ and the physical scalar boson interaction vertices are obtained by the substitution of the following expressions for $\bar\lambda_i$ and μ_i to (9):

$$\bar\lambda_1 = \frac{1}{2v^2}[(\tfrac{s_\alpha}{c_\beta})^2 m_h^2 + (\tfrac{c_\alpha}{c_\beta})^2 m_H^2 - \tfrac{s_\beta}{c_\beta^3}\mu_{12}^2] \tag{14}$$
$$\bar\lambda_2 = \frac{1}{2v^2}[(\tfrac{c_\alpha}{s_\beta})^2 m_h^2 + (\tfrac{s_\alpha}{s_\beta})^2 m_H^2 - \tfrac{c_\beta}{s_\beta^3}\mu_{12}^2] \tag{15}$$
$$\bar\lambda_3 = \frac{1}{v^2}[2m_{H^\pm}^2 - \tfrac{\mu_{12}^2}{s_\beta c_\beta} + \tfrac{s_{2\alpha}}{s_{2\beta}}(m_H^2 - m_h^2)] \tag{16}$$
$$\bar\lambda_4 = \frac{1}{v^2}(\tfrac{\mu_{12}^2}{s_\beta c_\beta} + m_A^2 - 2m_{H^\pm}^2) \tag{17}$$
$$\bar\lambda_5 = \frac{1}{v^2}(\tfrac{\mu_{12}^2}{s_\beta c_\beta} - m_A^2) \tag{18}$$
$$\mu_1^2 = \lambda_1 v_1^2 + (\bar\lambda_3 + \bar\lambda_4 + \bar\lambda_5)\tfrac{v_2^2}{2} - \mu_{12}^2\mathtt{tg}\beta \tag{19}$$
$$\mu_2^2 = \lambda_2 v_2^2 + (\bar\lambda_3 + \bar\lambda_4 + \bar\lambda_5)\tfrac{v_1^2}{2} - \mu_{12}^2\mathtt{ctg}\beta \tag{20}$$

Here (14)-(18) are the diagonalisation conditions and (19)-(20) are the minimization conditions. Two parametrisations for the Higgs boson self-interaction vertices can be used. In the first parametrisation [10] μ_{12}^2 is a free parameter and $\bar\lambda_5$ is defined by (18). In the second one $\bar\lambda_5$ is a free parameter and μ_{12}^2 is equal to $s_\beta c_\beta(v^2\bar\lambda_5 + m_A^2)$. Complete sets of Feynman rules (unitary gauge) for the triple (μ_{12}^2 and λ_5 parametrisations) and quartic (μ_{12}^2 parametrisation) Higgs boson interactions in the general two-Higgs-doublet model with CP-conservation are shown in Tables 1-2. [1]. In the case of MSSM potential $\bar\lambda_5$ =0 and it follows from (8),(10),(11) that μ_{12}^2 is fixed and equal to $m_A^2 s_\beta c_\beta$.

Two additional terms of the dimension 4 can be constructed using the complete set of $SU(2)\times U(1)$ invariants $\varphi_1^+\varphi_1$, $\varphi_2^+\varphi_2$, $\mathtt{Re}\varphi_1^+\varphi_2$ and $\mathtt{Im}\varphi_1^+\varphi_2$

[1] These sets were obtained by means of LanHEP package [11], see `http://theory.sinp.msu.ru/~semenov/lanhep.html`

(a detailed discussion of all possible potental forms can be found in [12]). These terms are usually added to the $U(\varphi_1,\varphi_2)$ with the couplings $\bar\lambda_6$ and $\bar\lambda_7$

$$\begin{aligned}\bar U(\varphi_1,\varphi_2) &= U(\varphi_1,\varphi_2) \\ &+\bar\lambda_6(\varphi_1^+\varphi_1)[(\varphi_1^+\varphi_2)+(\varphi_2^+\varphi_1)]+\bar\lambda_7(\varphi_2^+\varphi_2)[(\varphi_1^+\varphi_2)+(\varphi_2^+\varphi_1)]\end{aligned} \tag{21}$$

The diagonal form of $\bar U(\varphi_1,\varphi_2)$ at the same local minimum takes place at arbitrary μ_{12}^2, $\bar\lambda_6$, $\bar\lambda_7$ and can be achieved by means of the substitution with additional $\bar\lambda_6$, $\bar\lambda_7$ terms in the right-hand side:

$$\bar\lambda_1 = \frac{1}{2v^2}[(\frac{s_\alpha}{c_\beta})^2 m_h^2+(\frac{c_\alpha}{c_\beta})^2 m_H^2-\frac{s_\beta}{c_\beta^3}\mu_{12}^2]+\frac{1}{4}(\bar\lambda_7\mathtt{tg}^3\beta-3\bar\lambda_6\mathtt{tg}\beta) \tag{22}$$

$$\bar\lambda_2 = \frac{1}{2v^2}[(\frac{c_\alpha}{s_\beta})^2 m_h^2+(\frac{s_\alpha}{s_\beta})^2 m_H^2-\frac{c_\beta}{s_\beta^3}\mu_{12}^2]+\frac{1}{4}(\bar\lambda_6\mathtt{ctg}^3\beta-3\bar\lambda_7\mathtt{ctg}\beta) \tag{23}$$

$$\bar\lambda_3 = \frac{1}{v^2}[2m_{H^\pm}^2-\frac{\mu_{12}^2}{s_\beta c_\beta}+\frac{s_{2\alpha}}{s_{2\beta}}(m_H^2-m_h^2)]-\frac{\bar\lambda_6}{2}\mathtt{ctg}\beta-\frac{\bar\lambda_7}{2}\mathtt{tg}\beta \tag{24}$$

$$\bar\lambda_4 = \frac{1}{v^2}(\frac{\mu_{12}^2}{s_\beta c_\beta}+m_A^2-2m_{H^\pm}^2)-\frac{\bar\lambda_6}{2}\mathtt{ctg}\beta-\frac{\bar\lambda_7}{2}\mathtt{tg}\beta \tag{25}$$

$$\bar\lambda_5 = \frac{1}{v^2}(\frac{\mu_{12}^2}{s_\beta c_\beta}-m_A^2)-\frac{\bar\lambda_6}{2}\mathtt{ctg}\beta-\frac{\bar\lambda_7}{2}\mathtt{tg}\beta \tag{26}$$

$$\mu_1^2 = \bar\lambda_1 v_1^2+(\bar\lambda_3+\bar\lambda_4+\bar\lambda_5)\frac{v_2^2}{2}-\mu_{12}^2\mathtt{tg}\beta+\frac{v^2 s_\beta^2}{2}(3\bar\lambda_6\mathtt{ctg}\beta+\bar\lambda_7\mathtt{tg}\beta) \tag{27}$$

$$\mu_2^2 = \bar\lambda_2 v_2^2+(\bar\lambda_3+\bar\lambda_4+\bar\lambda_5)\frac{v_1^2}{2}-\mu_{12}^2\mathtt{ctg}\beta+\frac{v^2 c_\beta^2}{2}(\bar\lambda_6\mathtt{ctg}\beta+3\bar\lambda_7\mathtt{tg}\beta) \tag{28}$$

Our expressions for the redefined $\bar\lambda_4$ and $\bar\lambda_5$ are the same as given in [13].

The potentials (1) and (9) can be reduced to the MSSM potential in some regions of the parameter space which we are going to discuss. The potential $V(\varphi_1,\varphi_2)$ (1) has eight parameters: two vev's v_1, v_2 and six couplings λ_i (i=1,...6). Eight parameters of the potential $U(\varphi_1,\varphi_2)$ (9) μ_1, μ_2, μ_{12} and $\bar\lambda_i$ (i=1,...5) can be found using (10),(11). From the other side, in the Higgs sector we have eight physical parameters: the mixing angle β and W-boson mass m_W, mixing angle α, the parameter μ_{12} and four masses of scalars m_h, m_H, m_A, $m^\pm$. The m_W is fixed experimentally maintaining the constraint on the v_1, v_2, $v^2=v_1^2+v_2^2=4m_W^2/e^2\cdot\mathtt{sin}^2\theta_W$ which follows from the Higgs kinetic term $D_\mu\varphi D^\mu\varphi$ ($g=e/\mathtt{sin}\theta_W$, θ_W is the Weinberg angle). So the Higgs sector with the potentials (1) or (9) is described by a seven-dimensional parameter space. In the case of superpotential five additional constraints are imposed, relating all Higgs boson self-couplings $\bar\lambda_i$, (i=1,...5) to the gauge coupling constants at the energy scale M_{SUSY} [14]:

$$\bar{\lambda}_1^{SUSY} = \bar{\lambda}_2^{SUSY} = \frac{g^2+g_1^2}{8},\ \bar{\lambda}_3^{SUSY} = \frac{g^2-g_1^2}{4},\ \bar{\lambda}_4^{SUSY} = -\frac{g^2}{2},\ \bar{\lambda}_5^{SUSY} = 0. \tag{29}$$

The remaining two independent parameters may be used to define the four Higgs boson masses and two mixing angles. One can choose, for example, the r_1, r_2 parametrization [15] ($r_{1,2} = m^2_{h,H}/m^2_Z$) or the well-known m_A, $\mathtt{tg}\beta$ parametrization. In order to reduce the general two-Higgs-doublet model vertices to MSSM at the scale M_{SUSY} it is convenient to use the α, β parametrization:

$$m_h^2 = m_Z^2 c_{2\beta}\frac{s_{\alpha+\beta}}{s_{\alpha-\beta}}, \quad m_H^2 = m_Z^2 c_{2\beta}\frac{c_{\alpha+\beta}}{c_{\alpha-\beta}}, \quad m_A^2 = m_Z^2\frac{s_{2(\alpha+\beta)}}{s_{2(\alpha-\beta)}}, \quad \mu_{12}^2 = m_A^2 s_\beta c_\beta. \tag{30}$$

Substitution of these expressions to the vertex factors in Tables 1,2 after trivial trigonometric transformations reduces them to a simpler MSSM factors (see [4]). However, (30) are no longer valid at the energy scale m_W where the $\bar{\lambda}_i^{SUSY}$ couplings and masses of Higgs bosons are significantly changed by the radiative corrections and the effective two-Higgs-doublet potential should be described in the complete seven dimensional parameter space. Practical calculations of the radiatively corrected masses and/or couplings can be conveniently carried out using results of the two approaches, renormalization group (the HMSUSY package [16] or the analytical representation [17]) and diagrammatic (the FeynHiggsFast package, see [18]). Two different parametrizations can be used for these approaches.

In the RG approach it seems convenient to use the two-Higgs-doublet model parameter space m_A, $\mathtt{tg}\beta$, $\bar{\lambda}_1, ...\bar{\lambda}_5$. In the following we shall take into account the $\bar{\lambda}_6$ and $\bar{\lambda}_7$ terms defined in (15), so the parameter space will be nine-dimensional. RG evolution of the coupling constants λ_i from the energy scale M_{SUSY} to the electroweak scale m_W defines the λ_1,...λ_5 in (22)-(26) and the quartic couplings λ_6, λ_7. At a given m_A, $\mathtt{tg}\beta$, λ_6, λ_7 the parameter μ_{12}^2 and $m_{H^\pm}$ are fixed by the minimization conditions (25) and (26), the parameters μ_1^2 and μ_2^2 are fixed by (27) and (28), α can be calculated using (13), m_h and m_H can be found using the equations (22),(23). If we denote the deviation from the coupling $\bar{\lambda}_i^{SUSY}$ at the MSSM scale by $\Delta\bar{\lambda}_i$

$$2(\bar{\lambda}_{1,2}^{SUSY} - \bar{\lambda}_{1,2}) = \Delta\bar{\lambda}_{1,2}, \quad \bar{\lambda}_{3,4}^{SUSY} - \bar{\lambda}_{3,4} = \Delta\bar{\lambda}_{3,4}, \quad -\bar{\lambda}_{5,6,7} = \Delta\bar{\lambda}_{5,6,7}$$

we find the mixing angle (using (13) and introducing the notation $g_1^2 + g^2 = g^2\, m_Z^2/m_W^2$, $g^2 - g_1^2 = g^2(2 - \, m_Z^2/m_W^2)$)

$$\mathtt{tg}2\alpha = \frac{s_{2\beta}(m_A^2 + m_Z^2) + v^2((\Delta\bar{\lambda}_3 + \Delta\bar{\lambda}_4)s_{2\beta} + 2c_\beta^2\Delta\bar{\lambda}_6 + 2s_\beta^2\Delta\bar{\lambda}_7)}{c_{2\beta}(m_A^2 - m_Z^2) + v^2(\Delta\bar{\lambda}_1 c_\beta^2 - \Delta\bar{\lambda}_2 s_\beta^2 - \Delta\bar{\lambda}_5 c_{2\beta} + (\Delta\bar{\lambda}_6 - \Delta\bar{\lambda}_7)s_{2\beta})} \tag{31}$$

and CP-even Higgs boson masses (using the equations (14)-(18))

$$\begin{aligned}
m_H^2 &= c_{\alpha+\beta}^2 m_Z^2 + s_{\alpha-\beta}^2 m_A^2 \qquad (32)\\
&-v^2(\Delta\bar{\lambda}_1 c_\alpha^2 c_\beta^2 + \Delta\bar{\lambda}_2 s_\alpha^2 s_\beta^2 + 2(\Delta\bar{\lambda}_3 + \Delta\bar{\lambda}_4) c_\alpha c_\beta s_\alpha s_\beta + \Delta\bar{\lambda}_5 (c_\alpha^2 s_\beta^2 + s_\alpha^2 c_\beta^2))\\
&+2 s_{\alpha+\beta}(\Delta\bar{\lambda}_6 c_\alpha c_\beta + \Delta\bar{\lambda}_7 s_\alpha s_\beta)\\
m_h^2 &= s_{\alpha+\beta}^2 m_Z^2 + c_{\alpha-\beta}^2 m_A^2\\
&-v^2(\Delta\bar{\lambda}_1 s_\alpha^2 c_\beta^2 + \Delta\bar{\lambda}_2 c_\alpha^2 s_\beta^2 - 2(\Delta\bar{\lambda}_3 + \Delta\bar{\lambda}_4) c_\alpha c_\beta s_\alpha s_\beta + \Delta\bar{\lambda}_5 (s_\alpha^2 s_\beta^2 + c_\alpha^2 c_\beta^2))\\
&-2 c_{\alpha+\beta}(\Delta\bar{\lambda}_6 s_\alpha c_\beta - \Delta\bar{\lambda}_7 c_\alpha s_\beta)\\
m_{H^\pm}^2 &= m_W^2 + m_A^2 - \frac{v^2}{2}(\Delta\bar{\lambda}_5 - \Delta\bar{\lambda}_4)
\end{aligned}$$

with the minimization conditions

$$\begin{aligned}
\mu_{12}^2 &= s_\beta c_\beta [m_A^2 - \frac{v^2}{2}(2\Delta\bar{\lambda}_5 + \Delta\bar{\lambda}_6 \mathtt{ctg}\beta + \Delta\bar{\lambda}_7 \mathtt{tg}\beta)] \qquad (33)\\
\mu_1^2 &= \frac{1}{2} m_Z^2 c_{2\beta} - \mu_{12}^2 \mathtt{tg}\beta\\
&\quad -\frac{v^2}{2}[\Delta\bar{\lambda}_1 c_\beta^2 + (\Delta\bar{\lambda}_3 + \Delta\bar{\lambda}_4 + \Delta\bar{\lambda}_5) s_\beta^2 + 3\Delta\bar{\lambda}_6 s_\beta c_\beta + \Delta\bar{\lambda}_7 \frac{s_\beta^3}{c_\beta}]\\
\mu_2^2 &= -\frac{1}{2} m_Z^2 c_{2\beta} - \mu_{12}^2 \mathtt{ctg}\beta\\
&\quad -\frac{v^2}{2}[\Delta\bar{\lambda}_2 s_\beta^2 + (\Delta\bar{\lambda}_3 + \Delta\bar{\lambda}_4 + \Delta\bar{\lambda}_5) c_\beta^2 + \Delta\bar{\lambda}_6 \frac{c_\beta^3}{s_\beta} + 3\Delta\bar{\lambda}_7 s_\beta c_\beta]
\end{aligned}$$

These expressions can be straighforwardly used to calculate the radiatively corrected masses of Higgs bosons and the mixing angle α in the MSSM using a solution of the RG equations for $\bar{\lambda}_1, ...\bar{\lambda}_7$. Apparently, in the RG approach Feynman rules in terms of $\bar{\lambda}_i$ couplings are more convenient than rules in terms of Higgs particle masses.

In the diagrammatic approaches to calculation of the radiatively corrected masses [18] the corrections to m_h, m_H, m_A and $m_{H^\pm}$ are extracted from the renormalized Higgs boson self-energies (usually radiative corrections to only the CP-even Higgs boson masses are calculated). The set of 7+2 independent parameters inherent for the diagrammatic approaches could be m_A, $\mathtt{tg}\beta$, α, μ_{12}, m_h, m_H, $m_H^\pm$, and $\bar{\lambda}_6$, $\bar{\lambda}_7$. At a given m_A, $\mathtt{tg}\beta$, $\bar{\lambda}_6$, $\bar{\lambda}_7$ the μ_{12}^2 parameter can be fixed at the value $m_A^2 s_\beta c_\beta$, and α can be calculated using the renormalized self-energies correction [18] to the relation valid at the M_{SUSY} scale $m_A^2 + m_Z^2 = -s_{2\alpha}/s_{2\beta}(m_H^2 - m_h^2)$. Then $\bar{\lambda}_4$ is defined by (25) and $\bar{\lambda}_1$,...$\bar{\lambda}_3$ can be found using (22)-(24). In the diagrammatic calculations Feynman rules in terms of the radiatively corrected Higgs boson masses look more natural. Substitution of the radiatively corrected Higgs masses to Higgs vertex factors is expected to give results very close to those obtained from the loop corrections to Higgs vertex at the SUSY scale (see the discussion in the last ref. [8]). It has been shown in [19] on the example of hhh and $hhhh$ vertices (and for the case of diagonal third generation squark mass matrix) that large radiative corrections to the vertex factors calculated diagrammatically can be absorbed in the radiatively corrected Higgs boson masses.

Other parametrizations in the two-Higgs-doublet model are of course possible, but they should be carefully introduced to respect the minimization and diagonalisation conditions (22)-(28). The introduction of scalar particle masses and mixing angles inconsistent with them violates either diagonalisaton of the potential or its $SU(2)$ invariance, even if the minimization conditions remain valid.

3 CP-violation in the two-Higgs-doublet model

If complex vacuum expectation values of the ground state are taken in the potential (1), the complex coupling μ_{12}^2 in (9) straightforwardly appears from λ_5, λ_6 terms

$$\frac{\lambda_5}{4}[\varphi_1^+\varphi_2+\varphi_2^+\varphi_1-v_1v_2\mathtt{cos}\xi]^2+\frac{\lambda_6}{4}[-i(\varphi_1^+\varphi_2-\varphi_2^+\varphi_1)-v_1v_2\mathtt{sin}\xi]^2 \quad (34)$$
$$\Rightarrow (\frac{\lambda_5}{4}-\frac{\lambda_6}{4})[(\varphi_1^+\varphi_2)^2+(\varphi_2^+\varphi_1)^2]+(\frac{\lambda_5}{2}+\frac{\lambda_6}{2})\,\varphi_1^+\varphi_2\,\varphi_2^+\varphi_1$$
$$-\frac{v_1v_2}{2}(\lambda_5\mathtt{cos}\xi-i\lambda_6\mathtt{sin}\xi)\,\varphi_1^+\varphi_2-\frac{v_1v_2}{2}(\lambda_5\mathtt{cos}\xi+i\lambda_6\mathtt{sin}\xi)\,\varphi_2^+\varphi_1$$

so we find

$$\mu_{12}^2=\frac{v_1v_2}{2}(\lambda_5\mathtt{cos}\xi-i\lambda_6\mathtt{sin}\xi) \quad (35)$$

where $\lambda_6=2m_A^2/v^2$ (see (8)). With $\bar\lambda_6$ and $\bar\lambda_7$ terms (21) in the potential (9) the real and imaginary parts of the μ_{12}^2 parameter can be defined by the extension of the diagonalisation condition (26) to the $\bar\lambda_i$, μ complex plane

$$\mathtt{Re}\mu_{12}^2 \;=\; m_A^2s_\beta c_\beta+v^2(s_\beta c_\beta\mathtt{Re}\bar\lambda_5+\frac{1}{2}c_\beta^2\;\mathtt{Re}\bar\lambda_6+\frac{1}{2}s_\beta^2\;\mathtt{Re}\bar\lambda_7) \quad (36)$$
$$\mathtt{Im}\mu_{12}^2 \;=\; v^2(s_\beta c_\beta\mathtt{Im}\bar\lambda_5+\frac{1}{2}c_\beta^2\;\mathtt{Im}\bar\lambda_6+\frac{1}{2}s_\beta^2\;\mathtt{Im}\bar\lambda_7) \quad (37)$$

In the case of complex couplings μ_i and λ_i the effective potential (21) can be rewritten in the hermitian form

$$U(\varphi_1,\varphi_2)= \quad \tfrac{1}{2}[-\mu_1^2(\varphi_1^+\varphi_1)-\overset{*}{\mu_1^2}(\varphi_1^+\varphi_1)-\mu_2^2(\varphi_2^+\varphi_2)-\overset{*}{\mu_2^2}(\varphi_2^+\varphi_2)] \quad (38)$$

$$
\begin{aligned}
&-\mu_{12}^2(\varphi_1^+\varphi_2) - \overset{*}{\mu}{}_{12}^2(\varphi_2^+\varphi_1)\\
&+\tfrac{1}{2}[\bar\lambda_1(\varphi_1^+\varphi_1)^2 + \overset{*}{\bar\lambda}_1(\varphi_1^+\varphi_1)^2 + \bar\lambda_2(\varphi_2^+\varphi_2)^2 + \overset{*}{\bar\lambda}_2(\varphi_2^+\varphi_2)^2\\
&+\bar\lambda_3(\varphi_1^+\varphi_1)(\varphi_2^+\varphi_2) + \overset{*}{\bar\lambda}_3(\varphi_1^+\varphi_1)(\varphi_2^+\varphi_2)\\
&+\bar\lambda_4(\varphi_1^+\varphi_2)(\varphi_2^+\varphi_1) + \overset{*}{\bar\lambda}_4(\varphi_1^+\varphi_2)(\varphi_2^+\varphi_1)]\\
&+\tfrac{\bar\lambda_5}{2}(\varphi_1^+\varphi_2)(\varphi_1^+\varphi_2) + \tfrac{\overset{*}{\bar\lambda}_5}{2}(\varphi_2^+\varphi_1)(\varphi_2^+\varphi_1),\\
&+\bar\lambda_6(\varphi_1^+\varphi_1)(\varphi_1^+\varphi_2) + \overset{*}{\bar\lambda}_6(\varphi_1^+\varphi_1)(\varphi_2^+\varphi_1)\\
&+\bar\lambda_7(\varphi_2^+\varphi_2)(\varphi_1^+\varphi_2) + \overset{*}{\bar\lambda}_7(\varphi_2^+\varphi_2)(\varphi_2^+\varphi_1)
\end{aligned}
$$

$\overset{*}{a}$ denotes a complex conjugated. The substitution of complex μ_i and $\bar\lambda_i$ with the imaginary parts of μ_i and λ_i consistent with the extension of (22)-(28) to the potential (38) leads to the linear term and the non-diagonal mass term

$$
U(\varphi_1,\varphi_2) = c_0 A + c_1 hA + c_2 HA + \frac{m_h^2}{2}H^2 + \frac{m_H^2}{2}H^2 + \frac{m_A^2}{2}A^2 + m_{H^\pm}^2 H^{\pm 2} \quad (39)
$$
$$
+ \texttt{third and fourth order terms in } h, H, A, H^\pm
$$

where

$$
\begin{aligned}
c_0 &= -v\,\texttt{Im}\mu_{12}^2 + \frac{v^3}{2}s_\beta c_\beta \texttt{Im}\bar\lambda_5 + \frac{v^3}{2}(c_\beta^2\texttt{Im}\bar\lambda_6 + s_\beta^2\texttt{Im}\bar\lambda_7) \qquad (40)\\
c_1 &= -s_{\alpha-\beta}\,\texttt{Im}\mu_{12}^2 + \frac{v^2}{4}(s_{2\beta}s_{\alpha-\beta} - 2c_{\alpha+\beta})\texttt{Im}\bar\lambda_5\\
&\quad -\frac{v^2}{2}(s_\beta c_\beta c_{\alpha-\beta} - 3s_\alpha c_\beta)\texttt{Im}\bar\lambda_6 + \frac{v^2}{2}(s_\beta c_\beta c_{\alpha-\beta} - 3c_\alpha s_\beta)\texttt{Im}\bar\lambda_7\\
c_2 &= c_{\alpha-\beta}\,\texttt{Im}\mu_{12}^2 + \frac{v^2}{4}(c_{2\beta}s_{\alpha-\beta} - 3s_{\alpha+\beta})\texttt{Im}\bar\lambda_5\\
&\quad -\frac{v^2}{2}(c_\beta^2 c_{\alpha-\beta} + 2c_\alpha c_\beta)\texttt{Im}\bar\lambda_6 - \frac{v^2}{2}(s_\beta^2 c_{\alpha-\beta} + 2s_\alpha s_\beta)\texttt{Im}\bar\lambda_7
\end{aligned}
$$

In the CP-conserving limit s_ξ =0 the linear and non-diagonal second order terms hA and HA vanish. The linear term in A demonstrates that after the introduction of complex couplings we can be out of a local mimumum of the potential $\bar U(\varphi_1,\varphi_2)$. It is interesting to notice that the extension (37) of the diagonalisation condition for the imaginary parts $\texttt{Im}\mu_{12}^2$ and $\texttt{Im}\bar\lambda_5$ is compatible with the minimization condition (40) only if $\texttt{Im}\bar\lambda_5$ =0. If the potential has no $\bar\lambda_5$, $\bar\lambda_6$ and $\bar\lambda_7$ terms, the diagonal mass term exists only beyond the ground state. The restoration of the minimum in the latter case can be achieved by means of the opposite sign quantum correction term, originating from the tadpole diagrams with the pseudoscalar A connected to the squark loops [20] [2] . In the classical minimum c_0 =0 we find

$$\begin{aligned}
c_1 &= \frac{v^2}{2}(s_\alpha s_\beta^3 - c_\alpha c_\beta^3)\mathtt{Im}\bar{\lambda}_5 + v^2\,(s_\alpha c_\beta \mathtt{Im}\bar{\lambda}_6 - c_\alpha s_\beta \mathtt{Im}\bar{\lambda}_7) \\
c_2 &= -\frac{v^2}{2}(s_\alpha c_\beta^3 + c_\alpha s_\beta^3)\mathtt{Im}\bar{\lambda}_5 - v^2\,(c_\alpha c_\beta \mathtt{Im}\bar{\lambda}_6 + s_\alpha s_\beta \mathtt{Im}\bar{\lambda}_7)
\end{aligned} \tag{41}$$

The second order terms hA and HA in (39) can be removed as usual by the orthogonal rotation a_{ij} $(i,j$ =1,2,3) in h, H, A sector

$$(h, H, A)\; M^2 \begin{pmatrix} h \\ H \\ A \end{pmatrix} = (h_1, h_2, h_3)\; a^T_{ik}\; M^2_{kl}\; a_{lj} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} \tag{42}$$

where the mass matrix has the form

$$M^2 = \frac{1}{2}\begin{pmatrix} m_h^2 & 0 & c_1 \\ 0 & m_H^2 & c_2 \\ c_1 & c_2 & m_A^2 \end{pmatrix} \tag{43}$$

Squared masses of the physical states h_1, h_2, h_3, which are the Higgs bosons without definite CP-parity, are defined by the eigenvalues of mass matrix M^2 (roots of the cubic equation for eigenvalues are given by Cardano formulae)

$$\begin{aligned}
m^2_{h3} &= 2\sqrt{(-q)}\mathtt{cos}(\frac{\theta}{3} - \frac{a_2}{3}) \\
m^2_{h1} &= 2\sqrt{(-q)}\mathtt{cos}(\frac{\theta + 2\pi}{3} - \frac{a_2}{3}) \\
m^2_{h2} &= 2\sqrt{(-q)}\mathtt{cos}(\frac{\theta + 4\pi}{3} - \frac{a_2}{3})
\end{aligned} \tag{44}$$

where

$$\theta = \mathtt{arccos}\frac{r}{\sqrt{(-q^3)}}$$

$$r = \frac{1}{54}(9a_1 a_2 - 27 a_0 - 2a_2^3), \qquad q = \frac{1}{9}(3a_1 - a_2^2)$$

$$a_0 = c_1^2 m_H^2 + c_2^2 m_h^2 - m_h^2 m_H^2 m_A^2, \qquad a_1 = m_h^2 m_H^2 + m_h^2 m_A^2 + m_H^2 m_A^2 - c_1^2 - c_2^2,$$

$$a_2 = -m_h^2 - m_H^2 - m_A^2$$

[2]however, with nonzero λ_5, λ_6 and λ_7, the factor of the scalar-pseudoscalar Higgs counterterm is not explicitly proportional to the tadpole renormalization constant, or the tadpole parameter c_0.

One can see that in the limiting case of CP-conserving potential $\xi \to 0$ the following correspondence takes place: $m_{h_1} \to m_h$, $m_{h_2} \to m_H$ and $m_{h_3} \to m_A$. The normalized eigenvectors of the matrix M^2, which are at the same time the matrix elements of a_{ij}, $(h, H, A) = a_{ij} h_j$, have the form

$$a_{11} = \frac{1}{n_1}((m_H^2 - m_{h_1}^2)(m_A^2 - m_{h_1}^2) - c_2^2),\ a_{21} = \frac{1}{n_1} c_1 c_2,\ a_{31} = -\frac{1}{n_1} c_1 (m_H^2 - m_{h_1}^2)$$

$$a_{12} = \frac{1}{n_2} c_1 c_2,\ a_{22} = \frac{1}{n_2}((m_h^2 - m_{h_2}^2)(m_A^2 - m_{h_2}^2) - c_1^2),\ a_{32} = \frac{1}{n_2} c_2 (m_h^2 - m_{h_2}^2),$$

$$a_{13} = -\frac{1}{n_3} c_1 (m_H^2 - m_{h_3}^2),\ a_{23} = -\frac{1}{n_3} c_1 (m_H^2 - m_{h_3}^2),\ a_{33} = \frac{1}{n_3} (m_h^2 - m_{h_3}^2)(m_H^2 - m_{h_3}^2)$$

where $n_i = \sqrt{(}a_{1i}^2 + a_{2i}^2 + a_{3i}^2)$. Representations for the triple and quartic Higgs boson self-interactions in the case of CP-violating potential are given by the expansions of structures $a_{ij} h_j a_{ik} h_k a_{il} h_l$, and $a_{ij} h_j a_{ik} h_k a_{il} h_l a_{i,m} h_m$, they are bulky and not very telling, so we do not show them here. If the imaginary parts of $\bar{\lambda}_6$ and $\bar{\lambda}_7$ are not small, large off-diagonal elements of the mixing matrix a_{ij} could appear leading to significant mass splittings of the Higgs states and modifications of the Higgs boson interactions.

We assume that in the Yukawa sector $< \varphi_1 >$ couples only to down fermions

$$V_{ud} \frac{e m_d}{2\sqrt{2} m_W s_W c_\beta} [\bar{\psi}_1 (1 + \gamma_5) \psi_2 \varphi_1 + \bar{\psi}_2 (1 - \gamma_5) \psi_1 \varphi_1^+] \qquad (45)$$

where $(h, H, A) = a_{ij} h_j$, for the u, d quarks $\bar{\psi}_1 = \{\bar{u}, V_{ud}\bar{d} + V_{us}\bar{s} + V_{ub}\bar{b}\}$, $\quad \psi_2 = d$ and analogous structures for s,b quarks and leptons, in the case of quarks V_{ab} denotes the CKM matrix elements), and $< \varphi_2 >$ couples only to up fermions (model of type II [21]):

$$\frac{e m_u}{2\sqrt{2} m_W s_W s_\beta} [\bar{\psi}_1 (1 + \gamma_5) i\tau_2 \psi_2 \varphi_2^+ + \bar{\psi}_2 (1 - \gamma_5) i\tau_2 \psi_1 \varphi_2] \qquad (46)$$

where again physical h_1, h_2, h_3 states are introduced by means of the a_{ij} rotation, $\bar{\psi}_1 = \{\bar{u}, V_{ud}\bar{d} + V_{us}\bar{s} + V_{ub}\bar{b}\}$, $\quad \psi_2 = u$ and analogous structures for c and t quarks.

4 Higgs-gauge boson and Higgs-fermion couplings in the MSSM with explicit CP-violation

In the following we shall focus on the MSSM scenario for the two-Higgs-doublet model, which allows to restrict strongly the parameter space. The

SM-like scenarios in the general two-Higgs-doublet model have been discussed in [22]. Detailed consideration in the framework of MSSM has been performed in [5] (also [6]). In this section we would like only to compare qualitatively our results with the results of these approaches. Our calculation follows somewhat different scheme. In [5] the phase ξ of μ_{12}^2 is radiatively induced by the tadpole diagrams and can be absorbed in the definition of the μ parameter which appears in the stop mixing matrix off-diagonal element $A_t - \mu/\mathtt{tg}\beta$. (The $\bar\lambda_5$, $\bar\lambda_6$ and $\bar\lambda_7$ terms are also radiatively induced by the threshold effects.) At the same time the trilinear couplings A_t, A_b also carry a phase [3] , so both the radiatively induced and the trilinear phases contribute to the phase $\mathtt{arg}(\mu A)$ of the λ_6 and λ_7 terms. We do not account for the radiatively induced phase. In our calculation the phase ξ is independent physical parameter of the ground state of the general two-Higgs-doublet potential (38) with nonzero Born-level λ_i, $i=5,6,7$. The phase of μ_{12}^2 coupling depends on ξ, the phases of $\bar\lambda_5$, $\bar\lambda_6$ and $\bar\lambda_7$ are constrained by the minimization condition (40).

The couplings of W and Z bosons to the h_1, h_2, h_3 scalars have the form

$$
\begin{array}{ll}
V_\mu\, V_\nu\, h_1 & f_V g_{\mu\nu}(c_{\alpha-\beta}a_{21} - s_{\alpha-\beta}a_{11}) \\
V_\mu\, V_\nu\, h_2 & f_V g_{\mu\nu}(c_{\alpha-\beta}a_{22} - s_{\alpha-\beta}a_{12}) \\
V_\mu\, V_\nu\, h_3 & f_V g_{\mu\nu}(c_{\alpha-\beta}a_{23} - s_{\alpha-\beta}a_{13})
\end{array}
$$

where $V=W,Z$, $f_V = \frac{e}{s_W} m_W$ for the W and $f_V = \frac{e}{s_W c_W^2} m_W$ for the Z. The couplings of h_1, h_2, h_3 bosons to the t and b quarks have the form

$$
\begin{array}{ll}
\bar t\, t\, h_1 & f_t \frac{1}{s_\beta}(s_\alpha a_{21} + c_\alpha a_{11} - i c_\beta a_{31}\gamma_5) \\
\bar t\, t\, h_2 & f_t \frac{1}{s_\beta}(s_\alpha a_{22} + c_\alpha a_{12} - i c_\beta a_{32}\gamma_5) \\
\bar t\, t\, h_3 & f_t \frac{1}{s_\beta}(s_\alpha a_{23} + c_\alpha a_{13} - i c_\beta a_{33}\gamma_5) \\
 & \\
\bar b\, b\, h_1 & f_b \frac{1}{c_\beta}(c_\alpha a_{21} - s_\alpha a_{11} - i s_\beta a_{31}\gamma_5) \\
\bar b\, b\, h_2 & f_b \frac{1}{c_\beta}(c_\alpha a_{22} - s_\alpha a_{12} - i s_\beta a_{32}\gamma_5) \\
\bar b\, b\, h_3 & f_b \frac{1}{c_\beta}(c_\alpha a_{23} - s_\alpha a_{13} - i s_\beta a_{33}\gamma_5)
\end{array}
$$

where $f_{t,b} = -\frac{e}{2 s_W}\frac{m_{t,b}}{m_W}$.

In calculations of the $\bar\lambda_i$, we used the two-loop symbolic results for quartic couplings [17] obtained in the RG approach and extended to the case of CP-violation in [5]. The couplings $\bar\lambda_5$, $\bar\lambda_6$ and $\bar\lambda_7$ are nonzero and complex in the next-to-leading order approximmation (RG improved leading order approximation), so we can define the real and imaginary parts of μ_{12}^2 parameter

[3] for a rewiev see e.g. [23]

using (36) and (40)

$$\mathtt{Re}\mu_{12}^2 = m_A^2 s_\beta c_\beta + v^2(s_\beta c_\beta \mathtt{Re}\bar{\lambda}_5 + \frac{1}{2}c_\beta^2\ \mathtt{Re}\bar{\lambda}_6 + \frac{1}{2}s_\beta^2\ \mathtt{Re}\bar{\lambda}_7) \qquad (47)$$

$$\mathtt{Im}\mu_{12}^2 = \frac{v^2}{2}(s_\beta c_\beta \mathtt{Im}\bar{\lambda}_5 + c_\beta^2\ \mathtt{Im}\bar{\lambda}_6 + s_\beta^2\ \mathtt{Im}\bar{\lambda}_7) \qquad (48)$$

The Yukawa interaction of Higgs bosons with the third qeneration squarks (see e.g. [9]) involves the higgsino-neutralino μ-parameter and the trilinear parameters A_t, A_b which can be generally speaking complex. The $\bar{\lambda}_1, ...\bar{\lambda}_7$ couplings of [5] depend on the nine relevant parameters: μ, arg(μ), A_t, arg(A_t), A_b, arg(A_b), SUSY scale M_{SUSY}, m_A, $\mathtt{tg}\beta$. For the parameter set μ =3.5 TeV, $A_t = A_b$ =1.5 TeV, M_{SUSY} =0.5 TeV, m_A =220 GeV, $\mathtt{tg}\beta$ =4 which is typical for the region of MSSM parameter space where the imaginary parts of $\bar{\lambda}_6$ and $\bar{\lambda}_7$ are large (of the order of 1) [4] , we plot in Fig.2 the neutral Higgs boson masses and the mixing matrix elements as a function of $\bar{\lambda}_6$ and $\bar{\lambda}_7$ phase θ =arg(μA_t) =arg(μA_b). The Higgs boson mass spectrum of the CP-conserving limit θ =0 ($a_{ij} = diag\{1,1,1\}$) is shown in Fig.1. It is substantially changed when the phase θ is not small. The m_{h1} is smaller than the m_h and the m_{h_2} increases in comparison with a value in the CP-conserving limit. At the same time (see Fig.2) the h_1 couplings to gauge bosons W, Z decrease by about 15% if the phase of $\bar{\lambda}_6$, $\bar{\lambda}_7$ is large enough, while the h_2 couplings are not significantly changed. The changes of the $b\bar{b}h_1$ and the $b\bar{b}h_2$ coupling regime are also rather pronounced (Fig.3). In the region of MSSM parameter space where the m_{h_3} is around 150-250 GeV and the μ and $A_{t,b}$ parameters are of the order of TeV the regime of strong mixing in the Higgs sector takes place. As a result the light Higgs boson h_1 could have not been observed at LEP2 ($\sqrt{s}$ =200 GeV) for the reason of suppressed couplings to the gauge bosons, while the h_2, h_3 bosons are sufficiently heavy to be not produced on mass-shell at the LEP2 energy. Detailed analysis of this scenario can be found in [5, 6].

5 Triple and quartic Higgs boson couplings in the MSSM with explicit CP-violation

The $\bar{\lambda}_5$, $\bar{\lambda}_6$ and $\bar{\lambda}_7$ potential terms can modify significantly the Higgs boson self-interaction vertices calculated in the leading one-loop approximation. The $\bar{\lambda}_5$, $\bar{\lambda}_6$ and $\bar{\lambda}_7$ couplings of the next-to leading order approximation

[4]A detailed discussion of possible combined constraints on the MSSM parameter space from the cosmology, direct searches and indirect measurements (rare decays) can be found in [24]

include the terms $(-3/96\pi^2)h_t^4\mu^2 A_t^2/M_{SUSY}^4$ and $(-3/96\pi^2)h_t^4\mu(6A_t M_{SUSY}^2 - A_t^3)/M_{SUSY}^4$ (the Yukawa coupling $h_t \sim \sqrt{2}m_t/vs_\beta$) [5] which are of the order of one if the μ and A_t are taken at TeV energy scale. Looking for instance at the hhh vertex in the mass parametrization and taken in the CP-conserving limit ξ =0

$$
\begin{aligned}
g_{hhh} \;&=\; \frac{3e}{m_W s_W s_{2\beta}}[-(c_\beta c_\alpha^3 - s_\beta s_\alpha^3)m_h^2 + c_{\beta-\alpha}^2 c_{\beta+\alpha} m_A^2 \\
&\quad +c_{\beta-\alpha}^2(\bar{\lambda}_5 c_{\beta+\alpha} + \bar{\lambda}_6 c_\beta s_\alpha - \bar{\lambda}_7 s_\beta c_\alpha)v^2]
\end{aligned} \tag{49}
$$

we can observe that contributions of the λ terms and the mass terms in this case are of the same order. For the parameter set described in the previous section, we show the values of various triple and quartic Higgs boson self-interaction vertices as a function of the phase θ =arg(μA_t) =arg(μA_b) in Fig.4. The values of Higgs boson self-interaction vertices in the CP-conserving limit θ =0,π and in the leading order approximation $\bar{\lambda}_5 = \bar{\lambda}_6 = \bar{\lambda}_7$ =0 are marked in the same figures by horisontal arrows. The $\bar{\lambda}_5$, $\bar{\lambda}_6$ and $\bar{\lambda}_7$ potential terms induced in the next-to-leading order approximation introduce very large corrections to the triple and quartic self-interactions of Higgs bosons. In the region of the MSSM parameter space under consideration the difference of the leading order and the next-to-leading order vertex factors can be several times in some ranges of the phase variation.

6 Summary

We demonstrated the tree-level equivalence of the two-Higgs-doublet model potentials (1) and (9), where CP-invariance can be explicitly broken by the λ_6 term in (1) or by the complex μ_{12}^2 term in (9). The couplings λ_i (i=1,...6) of (1) and μ_1^2, μ_2^2, μ_{12}^2, $\bar{\lambda}_i$ (i=1,...5) of (9) are related by the equations (10). Diagonalisation of the potential (9) in the ground state (3) can be performed by means of the substitutions (14)-(20) which express the $\bar{\lambda}_i$ and μ_1^2, μ_2^2 couplings through the Higgs boson masses m_h, m_H, m_A, $m_{H\pm}$, the mixing angles α, β and the μ_{12}^2 parameter. In the general case the $\bar{\lambda}_6$ and $\bar{\lambda}_7$ potential terms (21) should be also considered with the diagonalisation and minimization conditions (22)-(28). If the complex couplings μ_1^2, μ_2^2, $\bar{\lambda}_i$ (i=1,...7) and the complex μ_{12}^2 parameter are introduced, the minimization of the hermitian Higgs potential (38) at the tree level takes place with the condition c_0 =0, see (40), for their imaginary parts. The imaginary parts of $\bar{\lambda}_5$, $\bar{\lambda}_6$ and $\bar{\lambda}_7$ couplings give rise to the CP-odd/CP-even Higgs boson off-diagonal terms, which are removed by the orthogonal rotation in (h, H, A) space, giving mass eigenstates h_1, h_2, h_3 without definite CP-parity and with the mass spectrum and couplings substantially different from the masses and couplings of CP-even and CP-odd states h, H, A, if the $\bar{\lambda}_5$, $\bar{\lambda}_6$ and $\bar{\lambda}_7$ are sufficiently large (of the order of one).

In the framework of MSSM the real parts of $\bar{\lambda}_i$ (i=1,...5) couplings are fixed at the SUSY energy scale by the conditions (29). Radiative corrections to the $\bar{\lambda}_i$ (i=1,...7) couplings are generated at the m_W energy scale. The equations (31)-(33) express the mixing angle α and masses of Higgs bosons in terms of the radiative corrections to $\bar{\lambda}_i^{SUSY}$ (i=1,...7) couplings (e.g. given by the RG evolution). They are valid independently on the particular scheme which is used for the calculation of radiative corrections to the $\bar{\lambda}_i^{SUSY}$ (i=1,...7).

In the next-to-leading order approximation the complex $\bar{\lambda}_5$, $\bar{\lambda}_6$ and $\bar{\lambda}_7$ couplings are generated by the Yukawa interaction of Higgs bosons with the third generation squarks. Using the results of [5] we calculated the Higgs-gauge boson, Higgs-fermion and the Higgs triple and quartic couplings for a representative SUSY parameter set, when the off-diagonal elements of the Higgs boson mixing matrix are large. The $\bar{\lambda}_5$, $\bar{\lambda}_6$ and $\bar{\lambda}_7$ couplings introduce significant corrections to the Higgs self-interaction, even in the case when their effects on the Higgs-gauge boson and Higgs-fermion couplings are rather small. These corrections could rather strongly (by one-two orders of magnitude in comparison with the case of CP-conservation) enhance or suppress some channels of multiple Higgs boson production at next colliders, providing discriminative tests of CP-violation in the Higgs sector and improved feasibility to reconstruct experimentally the Higgs potential.

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Fields in the vertex	Variational derivative of Lagrangian by fields
$h \quad h \quad h$	$\frac{3\,e}{M_W\,s_w\,s_{2\beta}^2}[-s_{2\beta}(c_\alpha^3 c_\beta - s_\alpha^3 s_\beta)m_h^2 + 2c_{\alpha-\beta}^2 c_{\alpha+\beta}\mu_{12}^2]$
$H \quad H \quad H$	$\frac{3\,e}{M_W s_w s_{2\beta}^2}[-s_{2\beta}(c_\alpha^3 s_\beta + s_\alpha^3 c_\beta)m_H^2 + 2s_{\alpha-\beta}^2 s_{\alpha+\beta}\mu_{12}^2]$
$H \quad H \quad h$	$\frac{e\,s_{\alpha-\beta}}{2M_W\,s_w\,s_{2\beta}^2}[-(2m_H^2 + m_h^2)s_{2\alpha}s_{2\beta} + 4(3s_\alpha c_\alpha + s_\beta c_\beta)\mu_{12}^2]$
$H \quad h \quad h$	$-\frac{e\,c_{\alpha-\beta}}{2M_W\,s_w\,s_{2\beta}^2}[(m_H^2 + 2m_h^2)s_{2\alpha}s_{2\beta} - 4(3s_\alpha c_\alpha - s_\beta c_\beta)\mu_{12}^2]$
$H \quad A \quad A$	$-\frac{e}{M_W\,s_w\,s_{2\beta}^2}[s_{2\beta}(s_\alpha c_\beta^3 + c_\alpha s_\beta^3)m_H^2 + s_{2\beta}^2 c_{\alpha-\beta}m_A^2 - 2s_{\alpha+\beta}\mu_{12}^2]$
$h \quad A \quad A$	$\frac{e}{M_W\,s_w\,s_{2\beta}^2}[s_{2\beta}(s_\alpha s_\beta^3 - c_\alpha c_\beta^3)m_h^2 + s_{2\beta}^2 s_{\alpha-\beta}m_A^2 + 2c_{\alpha+\beta}\mu_{12}^2]$
$h \quad H^+ \quad H^-$	$\frac{e}{M_W\,s_w\,s_{2\beta}^2}[s_{2\beta}(s_\alpha s_\beta^3 - c_\alpha c_\beta^3)m_h^2 + s_{2\beta}^2 s_{\alpha-\beta}m_{H^\pm}^2 + 2c_{\alpha+\beta}\mu_{12}^2]$
$H \quad H^+ \quad H^-$	$-\frac{e}{M_W\,s_w\,s_{2\beta}^2}[s_{2\beta}(s_\alpha c_\beta^3 + c_\alpha s_\beta^3)m_H^2 + s_{2\beta}^2 c_{\alpha-\beta}m_{H^\pm}^2 - 2s_{\alpha+\beta}\mu_{12}^2]$

Table 1. Triple Higgs boson interaction vertices in the general two-Higgs-doublet model, μ_{12} parametrisation.

Fields in the vertex	Variational derivative of Lagrangian by fields
$h \quad h \quad h$	$\frac{3\,e}{M_W\,s_w\,s_{2\beta}}[-(c_\alpha^3 c_\beta - s_\alpha^3 s_\beta)m_h^2 + c_{\alpha-\beta}^2 c_{\alpha+\beta}(m_A^2 + v^2\lambda_5)]$
$H \quad H \quad H$	$\frac{3\,e}{M_W s_w s_{2\beta}}[-(c_\alpha^3 s_\beta + s_\alpha^3 c_\beta)m_H^2 + s_{\alpha-\beta}^2 s_{\alpha+\beta}(m_A^2 + v^2\lambda_5)]$
$H \quad H \quad h$	$\frac{e\,s_{\alpha-\beta}}{2M_W\,s_w\,s_{2\beta}}[-(2m_H^2 + m_h^2)s_{2\alpha} + 2(3s_\alpha c_\alpha + s_\beta c_\beta)(m_A^2 + v^2\lambda_5)]$
$H \quad h \quad h$	$-\frac{e\,c_{\alpha-\beta}}{2M_W\,s_w\,s_{2\beta}}[(m_H^2 + 2m_h^2)s_{2\alpha} - 2(3s_\alpha c_\alpha - s_\beta c_\beta)(m_A^2 + v^2\lambda_5)]$
$H \quad A \quad A$	$-\frac{e}{M_W\,s_w\,s_{2\beta}}[(s_\alpha c_\beta^3 + c_\alpha s_\beta^3)m_H^2 + c_{2\beta}s_{\alpha-\beta}m_A^2 - s_{\alpha+\beta}v^2\lambda_5]$
$h \quad A \quad A$	$\frac{e}{M_W\,s_w\,s_{2\beta}}[(s_\alpha s_\beta^3 - c_\alpha c_\beta^3)m_h^2 + c_{2\beta}c_{\alpha-\beta}m_A^2 + c_{\alpha+\beta}v^2\lambda_5]$
$h \quad H^+ \quad H^-$	$\frac{e}{M_W\,s_w\,s_{2\beta}}[(s_\alpha s_\beta^3 - c_\alpha c_\beta^3)m_h^2 + s_{\alpha-\beta}m_{H^\pm}^2 + c_{\alpha+\beta}(m_A^2 + v^2\lambda_5)]$
$H \quad H^+ \quad H^-$	$-\frac{e}{M_W\,s_w\,s_{2\beta}}[(s_\alpha c_\beta^3 + c_\alpha s_\beta^3)m_H^2 + c_{\alpha-\beta}m_{H^\pm}^2 - s_{\alpha+\beta}(m_A^2 + v^2\lambda_5)]$

Table 2. Triple Higgs boson interaction vertices in the general two-Higgs-doublet model, λ_5 parametrisation.

Fields in the vertex	Variational derivative of Lagrangian by fields
$h\quad h\quad h\quad h$	$-\frac{3}{4}\frac{e^2}{M_W^2\,s_w^2\,s_{2\beta}^3}[4s_{2\beta}(c_\alpha^3 c_\beta+s_\alpha^3 s_\beta)^2 m_h^2+s_{2\beta}s_{2\alpha}^2 c_{\alpha-\beta}^2 m_H^2-8c_{\alpha-\beta}^2 c_{\alpha+\beta}^2\mu_{12}^2]$
$H\quad H\quad H\quad H$	$\frac{3}{4}\frac{e^2}{M_W^2\,s_w^2\,s_{2\beta}^3}[-4s_{2\beta}(c_\alpha^3 s_\beta+s_\alpha^3 c_\beta)^2 m_H^2+s_{2\beta}s_{2\alpha}^2 s_{\alpha-\beta}^2 m_h^2+8s_{\alpha-\beta}^2 s_{\alpha+\beta}^2\mu_{12}^2]$
$A^0\quad A^0\quad A^0\quad A^0$	$-3\frac{e^2}{M_W^2\,s_w^2\,s_{2\beta}^3}[s_{2\beta}(s_\alpha c_\beta^3+c_\alpha s_\beta^3)^2 m_H^2 s_{2\beta}(c_\alpha c_\beta^3+s_\alpha s_\beta^3)^2 m_h^2-2c_{2\beta}^2\mu_{12}^2]$
$H\quad H\quad H\quad h$	$-\frac{3}{4}\frac{e^2 s_{2\alpha}s_{\alpha-\beta}}{M_W^2\,s_w^2\,s_{2\beta}^3}[2s_{2\beta}(c_\alpha^3 s_\beta+s_\alpha^3 c_\beta)m_H^2+s_{2\beta}s_{2\alpha}c_{\alpha-\beta}m_h^2-4s_{\alpha+\beta}]\mu_{12}^2$
$H\quad h\quad h\quad h$	$-\frac{3}{4}\frac{e^2 s_{2\alpha}c_{\alpha-\beta}}{M_W^2\,s_w^2\,s_{2\beta}^3}[2s_{2\beta}(c_\alpha^3 c_\beta-s_\alpha^3 s_\beta)m_h^2+s_{2\beta}s_{2\alpha}s_{\alpha-\beta}m_H^2-4c_{\alpha+\beta}]\mu_{12}^2$
$H\quad H\quad h\quad h$	$-\frac{1}{4}\frac{e^2}{M_W^2\,s_w^2\,s_{2\beta}^3}[-s_{2\beta}s_{2\alpha}(3s_{2\alpha}s_{\alpha-\beta}^2-4s_{\alpha-\beta}c_{\alpha+\beta}-2s_{\alpha+\beta}s_{\alpha-\beta}c_{\alpha-\beta})m_h^2$ $+s_{2\beta}s_{2\alpha}(s_{2\beta}+3s_{2\alpha}s_{\alpha-\beta}^2)m_H^2-8(3s_\alpha^2c_\alpha^2-s_\beta^2c_\beta^2)\mu_{12}^2]$
$H\quad H\quad A^0\quad A^0$	$\frac{1}{4}\frac{e^2}{M_W^2\,s_w^2\,s_{2\beta}^3}[-2s_{2\beta}s_{2\alpha}s_{\alpha-\beta}(c_\alpha c_\beta^3-s_\alpha s_\beta^3)m_h^2-2s_{2\beta}^3c_{\alpha-\beta}^2m_A^2$ $-s_{2\beta}(s_{2\alpha}s_{2\beta}+3s_{\alpha-\beta}^2s_{\alpha+\beta}^2-s_{2\beta}s_{\alpha-\beta}^2)m_H^2+4(c_{2\beta}^2s_{\alpha-\beta}^2+s_{\alpha+\beta}^2)\mu_{12}^2]$
$h\quad h\quad A^0\quad A^0$	$\frac{1}{4}\frac{e^2}{M_W^2\,s_w^2\,s_{2\beta}^3}[-s_{2\beta}^2(4c_{2\beta}c_{2\alpha}+3s_{\alpha-\beta}^2s_{\alpha+\beta}^2+s_{\alpha-\beta}^4)m_h^2-2s_{2\beta}^3s_{\alpha-\beta}^2m_A^2$ $-2s_{2\beta}s_{2\alpha}c_{\alpha-\beta}(s_\alpha c_\beta^3+c_\alpha s_\beta^3)m_H^2+2(s_{2\beta}^2s_{\alpha-\beta}^2+4(c_\alpha c_\beta^3+s_\alpha s_\beta^3)^2)\mu_{12}^2]$
$H\quad A^0\quad A^0\quad h$	$\frac{1}{4}\frac{e^2}{M_W^2\,s_w^2\,s_{2\beta}^3}[-2s_{2\beta}s_{2\alpha}c_{\alpha-\beta}(c_\alpha c_\beta^3-s_\alpha s_\beta^3)m_h^2+s_{2\beta}^3s_{\alpha-\beta}c_{\alpha-\beta}m_A^2$ $-2s_{2\beta}s_{2\alpha}s_{\alpha-\beta}(s_\alpha c_\beta^3+c_\alpha s_\beta^3)m_H^2+2(2s_{2\alpha}c_{2\beta}-s_{2\beta}s_{\alpha-\beta}c_{\alpha-\beta})\mu_{12}^2]$
$H^+\quad H^+\quad H^-\quad H^-$	$-2\frac{e^2}{M_W^2\,s_w^2\,s_{2\beta}^3}[s_{2\beta}(c_\alpha c_\beta^3+s_\alpha s_\beta^3)m_h^2+s_{2\beta}(s_\alpha c_\beta^3+c_\alpha s_\beta^3)m_H^2-2c_{2\beta}^2\mu_{12}^2]$
$H^+\quad H^-\quad A^0\quad A^0$	$-\frac{e^2}{M_W^2\,s_w^2\,s_{2\beta}^3}[s_{2\beta}(c_\alpha c_\beta^3+s_\alpha s_\beta^3)m_h^2+s_{2\beta}(s_\alpha c_\beta^3+c_\alpha s_\beta^3)m_H^2-2c_{2\beta}^2\mu_{12}^2]$
$H^+\quad H^-\quad h\quad h$	$\frac{1}{4}\frac{e^2}{M_W^2\,s_w^2\,s_{2\beta}^3}[-s_{2\beta}(4c_{2\alpha}c_{2\beta}+3s_{\alpha-\beta}^2s_{\alpha+\beta}^2+s_{\alpha-\beta}^4)m_h^2-2s_{2\beta}^3s_{\alpha-\beta}^2m_{H^\pm}^2$ $-2s_{2\beta}s_{2\alpha}c_{\alpha-\beta}(s_\alpha c_\beta^3+c_\alpha s_\beta^3)m_H^2+2(s_{2\beta}^2s_{\alpha-\beta}^2+4(c_\alpha c_\beta^3+s_\alpha s_\beta^3)^2\mu_{12}^2]$
$H^+\quad H^-\quad H\quad H$	$\frac{1}{4}\frac{e^2}{M_W^2\,s_w^2\,s_{2\beta}^3}[-2s_{2\beta}s_{2\alpha}s_{\alpha-\beta}(c_\alpha c_\beta^3-s_\alpha s_\beta^3)m_h^2-2s_{2\beta}^3c_{\alpha-\beta}^2m_{H^\pm}^2$ $+s_{2\beta}(s_{2\alpha}s_{2\beta}-3s_{\alpha-\beta}^2s_{\alpha+\beta}^2+s_{\alpha-\beta}^4)m_H^2+4(c_{2\beta}^2s_{\alpha-\beta}^2+s_{\alpha+\beta}^2)\mu_{12}^2]$
$H\quad H^+\quad H^-\quad h$	$\frac{1}{2}\frac{e^2}{M_W^2\,s_w^2\,s_{2\beta}^3}[-s_{2\beta}s_{2\alpha}c_{\alpha-\beta}(c_\alpha c_\beta^3-s_\alpha s_\beta^3)m_h^2+s_{2\beta}^3s_{\alpha-\beta}c_{\alpha-\beta}m_{H^\pm}^2$ $-s_{2\beta}s_{2\alpha}s_{\alpha-\beta}(c_\alpha s_\beta^3+s_\alpha c_\beta^3)m_H^2+2(2s_{2\alpha}c_{2\beta}-s_{2\beta}^2s_{\alpha-\beta}c_{\alpha-\beta})\mu_{12}^2]$

Table 3. Quartic Higgs boson interaction vertices in the general two-Higgs-doublet model, μ_{12} parametrisation.

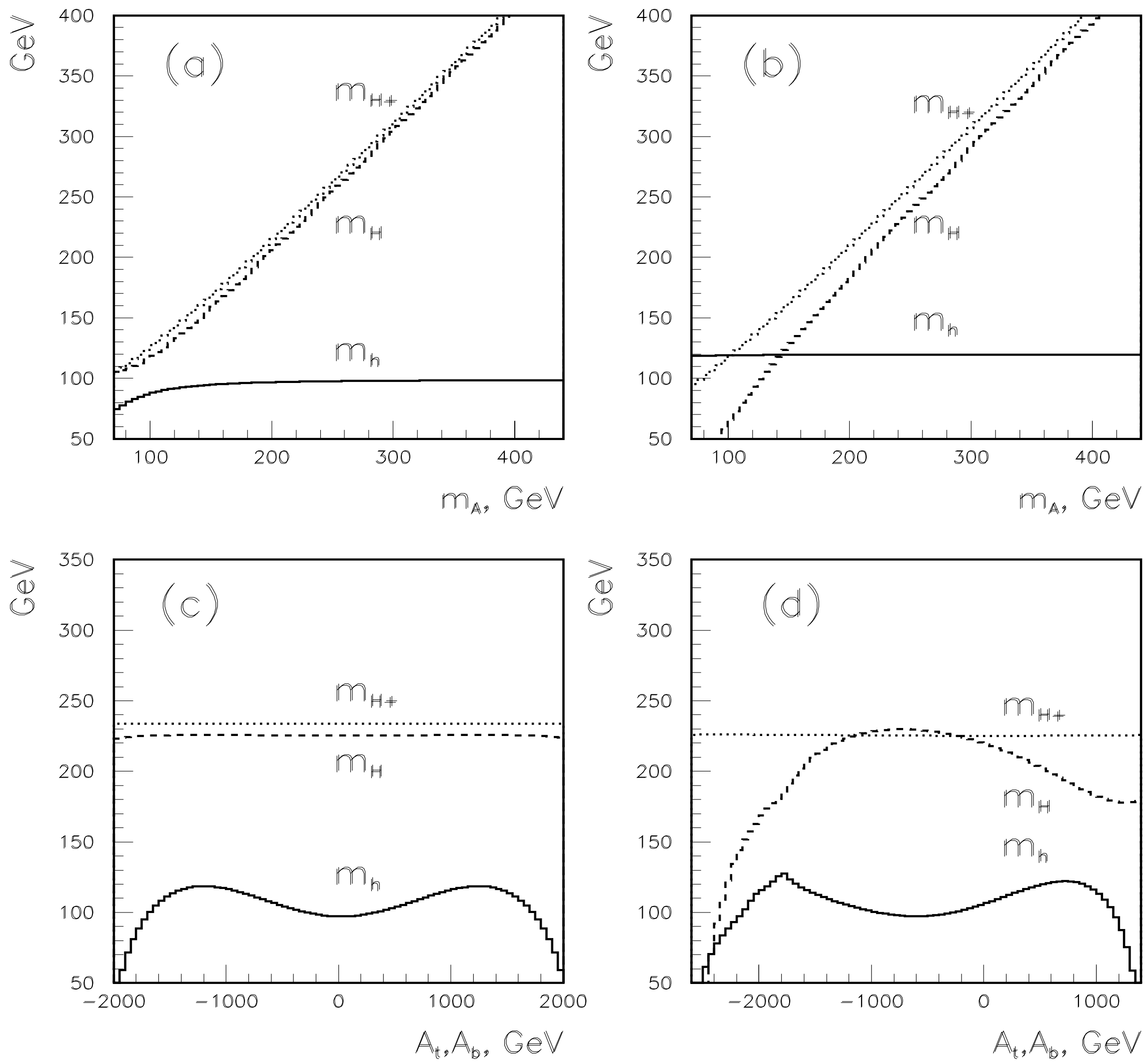

Figure 1: Masses of the neutral and charged Higgs bosons $h, H, H^{\pm}$ versus the pseudoscalar mass m_A and the trilinear constants A_t, A_b calculated by means of (31),(32) with the analytical $\bar{\lambda}_i$ (i=1,...7) parametrization of [5]. The $\Delta\lambda_5$ is chosen to be positive. The CP-conserving limit ξ =0 is taken. (a) $\mathtt{tg}\beta$ =4, M_{SUSY} =0.5 TeV, $A_t = A_b = \mu$ =0; (b) $\mathtt{tg}\beta$ =4, M_{SUSY} =0.5 TeV, $A_t = A_b$ =0.9 TeV, $\mu = -1.5$ TeV; (c) $\mathtt{tg}\beta$ =4, M_{SUSY} =0.5 TeV, m_A =220 GeV, μ =0, $A_t = A_b$; (d) $\mathtt{tg}\beta$ =4, M_{SUSY} =0.5 TeV, m_A =220 GeV, $\mu = -2$ TeV, $A_t = A_b$. Very small variations of the charged Higgs boson mass $m_{H^{\pm}}$ in (d) are due to the cancellation of leading power terms $\sim \mu^2 A_{t,b}^2/M_{SUSY}^4$, see [5], in the difference of $\bar{\lambda}_4$ and $\bar{\lambda}_5$, see (32). If $\Delta\lambda_5$ is chosen to be negative, m_H increases in comparison with the case $A_t = A_b = \mu$ =0.

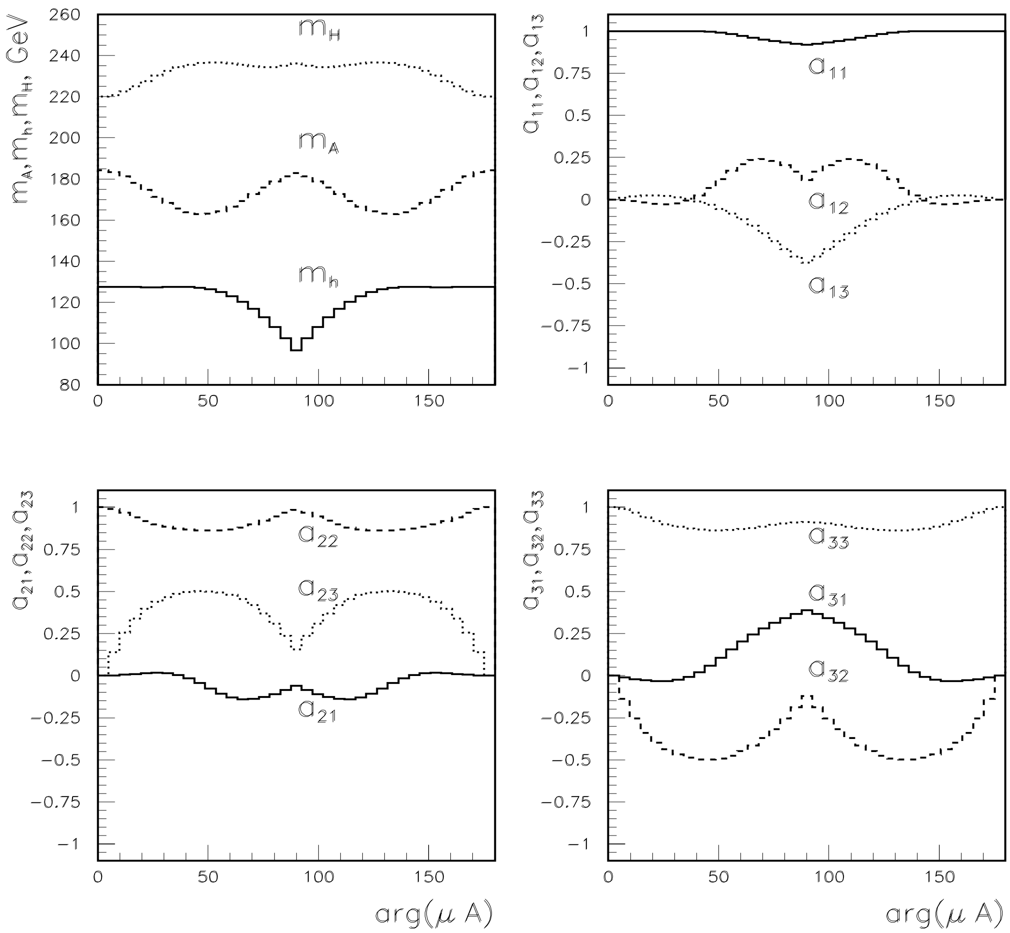

Figure 2: Masses of the neutral Higgs bosons and the mixing matrix elements as a function of the $\bar{\lambda}_6$ and $\bar{\lambda}_7$ phase. The $\bar{\lambda}_i$ couplings are taken from [5] at the parameter values $\mathtt{tg}\beta = 4$, $m_A = 220$ GeV, $M_{SUSY} = 0.5$ TeV, $A_t = A_b = -1.8$ TeV, $\mu = -2$ TeV.

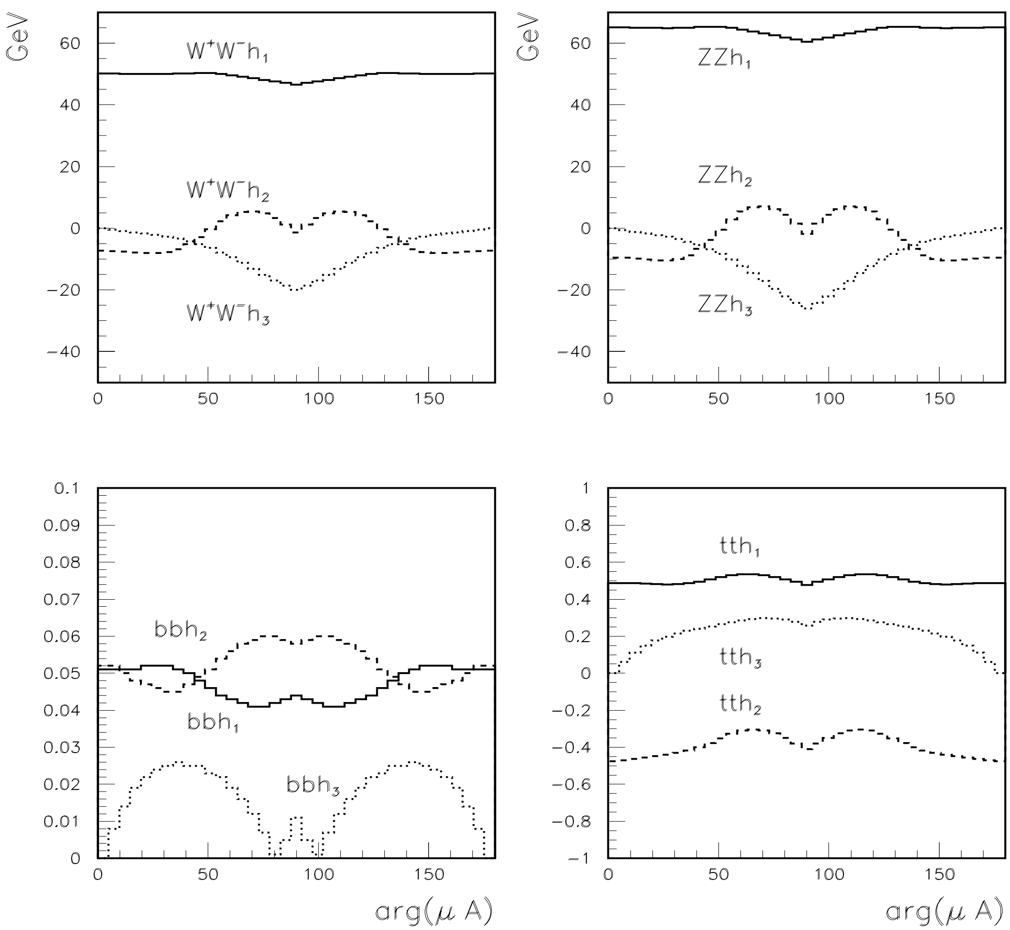

Figure 3: Higgs-gauge boson and Higgs-fermion vertex factors as a function of the $\bar{\lambda}_6$ and $\bar{\lambda}_7$ phase. The $\bar{\lambda}_i$ couplings are taken from [5] at the parameter values $\mathtt{tg}\beta$ =4, m_A =220 GeV, M_{SUSY} =0.5 TeV, $A_t = A_b = -1.8$ TeV, $\mu = -2$ TeV. For the coupling with fermions we plot $\sqrt{\mathtt{Im}^2 g_{ffh} + \mathtt{Re}^2 g_{ffh}}$.

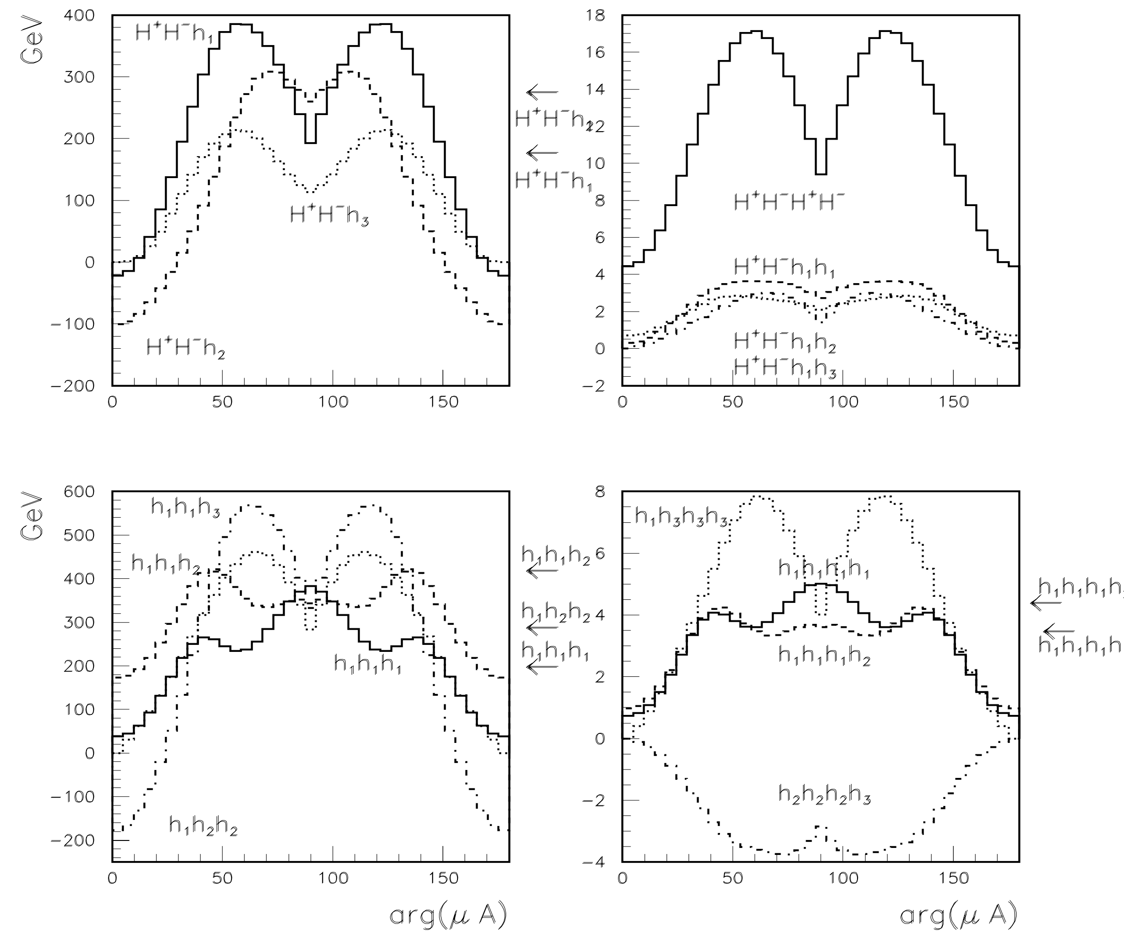

Figure 4: Triple and quartic Higgs boson vertex factors as a function of the $\bar{\lambda}_6$ and $\bar{\lambda}_7$ phase. The $\bar{\lambda}_i$ couplings are taken from [5] at the parameter values $\mathtt{tg}\beta$ =4, m_A =220 GeV, M_{SUSY} =0.5 TeV, $A_t = A_b = -1.8$ TeV, $\mu = -2$ TeV. Horisontal arrows indicate the values of vertex factors in the CP-conserving limit ξ =0 and the leading order approximation $\bar{\lambda}_5 = \bar{\lambda}_6 = \bar{\lambda}_7$ =0.